2019 **HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen, black is preferred
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6 – 11)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 to 10.

---

1. Which of the following is equivalent to $\tan\left(\frac{\pi}{4}+x\right)$?

(A) $\dfrac{\sec^2 x}{1-\tan^2 x}$

(B) $\dfrac{\cos x - \sin x}{\cos x + \sin x}$

(C) $\dfrac{\sin x + \cos x}{\sin x - \cos x}$

(D) $\dfrac{\cos x + \sin x}{\cos x - \sin x}$

2. What is the Cartesian equation of the tangent to the parabola $x = t - 3, y = t^2 + 2$ at $t = -3$?

(A) $6x - y - 25 = 0$

(B) $6x + y + 25 = 0$

(C) $6x + y + 36 = 0$

(D) $6x + 2y - 25 = 0$

3. After t years the number N of individuals in a population is given by $N = 400 + 100e^{-0 \cdot 1t}$.
What is the difference between the initial population size and the limiting population size?

(A) 100

(B) 300

(C) 400

(D) 500

4. The expression $\sin x - \sqrt{3}\cos x$ can be written in the form $2\sin(x + \alpha)$.

What is the value of α?

(A) $-\dfrac{\pi}{3}$

(B) $-\dfrac{\pi}{6}$

(C) $\dfrac{\pi}{6}$

(D) $\dfrac{\pi}{3}$

5. What is the coefficient of x^5 in $\left(x^2 + \dfrac{2}{x}\right)^7$?

(A) ${}^7\mathbf{C}_3 \times 2^3$

(B) ${}^7\mathbf{C}_4 \times 2^4$

(C) ${}^7\mathbf{C}_5 \times 2^5$

(D) ${}^7\mathbf{C}_4 \times 2^7$

6. What is the domain and range of $y = 3\sin^{-1}(2x)$?

(A) Domain: $-\dfrac{1}{2} \le x \le \dfrac{1}{2}$, Range: $-\dfrac{1}{3} \le y \le \dfrac{1}{3}$

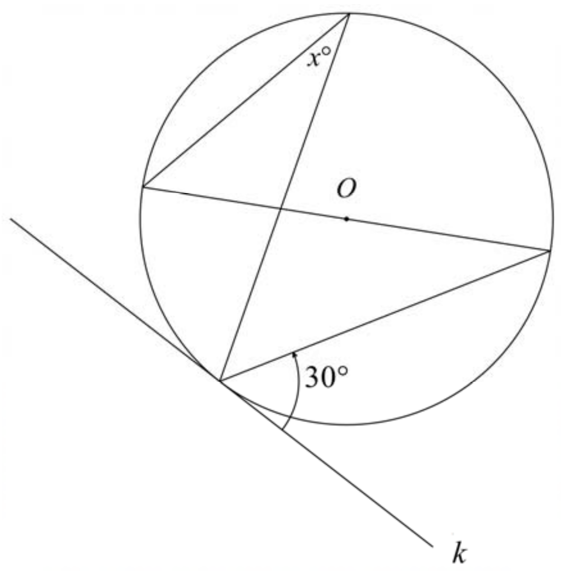
(B) Domain: $-\dfrac{1}{2} \le x \le \dfrac{1}{2}$, Range: $-\dfrac{3\pi}{2} \le y \le \dfrac{3\pi}{2}$

(C) Domain: $-2 \le x \le 2$, Range: $-\dfrac{1}{3} \le y \le \dfrac{1}{3}$

(D) Domain: $-2 \le x \le 2$, Range: $-\dfrac{3\pi}{2} \le y \le \dfrac{3\pi}{2}$

7. O is the centre of a circle. The line k is tangent to the circle.

What is the value of x?



(A) 30°

(B) 45°

(C) 60°

(D) 90°

8. A particle is moving in simple harmonic motion about the origin according to the equation $x = 2\cos(nt)$, where x is its displacement from the origin after t seconds. It passes through the origin with speed $\sqrt{2}$ m/s.

What is the value of n?

(A) $-\sqrt{2}$

(B) $\dfrac{1}{\sqrt{2}}$

(C) $\dfrac{\pi}{4}$

(D) $\sqrt{2}$

9. How many solutions does the equation $\sin^2 x + 3\sin x\cos x + 2\cos^2 x = 0$ have in the domain $0 \leq x \leq 2\pi$?

(A) 1

(B) 2

(C) 3

(D) 4

10. By using symmetry arguments, what is the value of $\int_{-a}^{a} \cos^{-1} x\,dx$ where $-1 \leq a \leq 1$?

(A) 0

(B) $\frac{a\pi}{2}$

(C) $a\pi$

(D) $2a\pi$

Section II

60 marks
Attempt questions 11 -14
Allow about 1 hour 45 minutes for this section

Answer each question on the appropriate page of your answer booklet
In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Answer on the appropriate page

(a) Evaluate $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$, giving your answer in exact form. **2**

(b) The point A is $(-2,1)$ and the point B is $(1,5)$. Find the coordinates of the point Q which divides AB externally in the ratio 5:2. **2**

(c) Solve $\tan\theta = \sin 2\theta$ for $0 \le \theta \le \pi$. **3**

(d) Differentiate $y = \ln(\tan^{-1} 2x)$. **3**

(e) For what values of x, where $x \neq 0$, does the geometric series **3**

$1 + \frac{2x}{x+1} + \left(\frac{2x}{x+1}\right)^2 + \ldots$ have a limiting sum?

(f) Find the general solution of $\sqrt{3}\tan 3\theta = 1$. **2**

End of Question 11

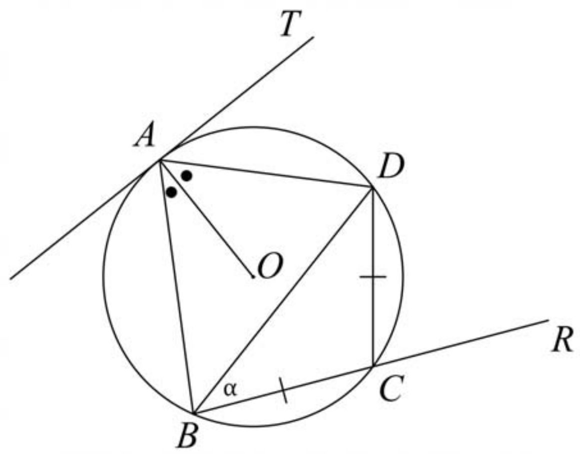
Question 12 (15 marks) Answer on the appropriate page

(a) Find the acute angle (to the nearest degree) between the lines $3x - 8y - 7 = 0$ and $y = -2x + 5$. **2**

(b) Use the substitution $u = 2 + x^2$ to find $\int_1^2 x\sqrt{2 + x^2}\,dx$. **3**

(c) If α, β and γ are the roots of the equation $x^3 - 2x^2 + 4x + 7 = 0$, find the value of $(2\alpha + 1)(2\beta + 1)(2\gamma + 1)$. **2**

(d) The points *A, B, C* and *D* are points on the circumference of a circle with centre at *O.* The line *TA* is a tangent to the circle at *A* and *BC* is produced to *R.* The interval *OA* bisects $\angle BAD$ and *BC*=*CD.* The size of $\angle DBC$ is α.



(i) Copy the diagram into your answer booklet and explain why $\angle DCR = 2\alpha$. **1**

(ii) Show that $\angle OAD = \alpha$. **1**

(iii) Prove that $\angle ABC$ is a right angle. **2**

(e) (i) Sketch the graph of $y = 2\sin x$ for $0 \le x \le \pi$. **1**

(ii) On the same set of axes sketch $x - 3y = 0$. **1**

(iii) Assuming the approximate point of intersection of the two graphs is at *x* =2.7, use one application of Newton's method to find a second approximation. (Answer correct to 3 decimal places). **2**

End of Question 12

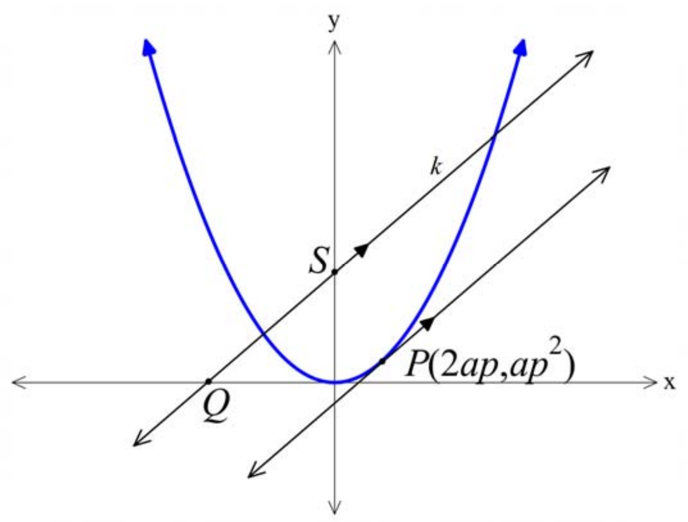
Question 13 (15 marks) Answer on the appropriate page

(a) Let $f(x) = 5 - \sqrt{x}$.

(i) Sketch the inverse function $y = f^{-1}(x)$. **1**

(ii) Find the equation of the inverse function $y = f^{-1}(x)$. **2**

(b) The point $P(2ap, ap^2)$, where $p \neq 0$, is a point on the parabola $x^2 = 4ay$. The line k is parallel to the tangent at P and passes through the focus, S, of the parabola.



(i) Find the equation of the line k. **2**

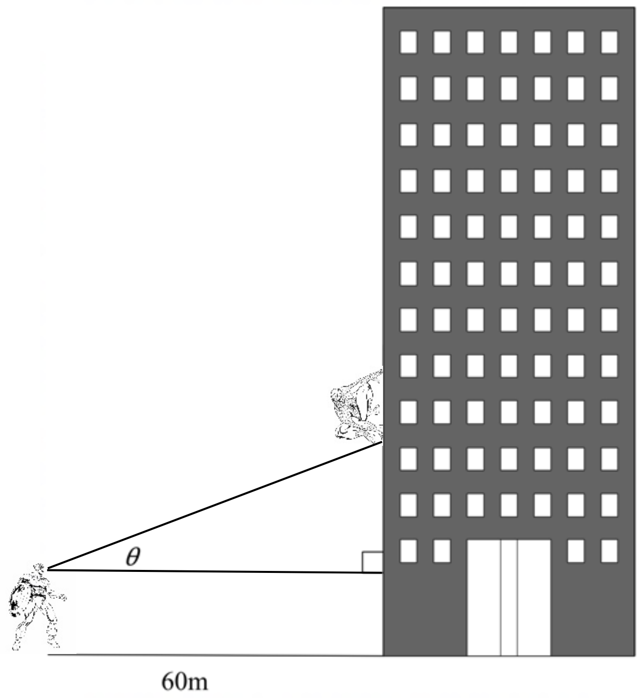
(ii) The line k intersects the x axis at the point Q. Find the coordinates of the midpoint, M, of the interval QS. **2**

(iii) Find the equation of the locus of M. **1**

Question 13 continues on the following page

Question 13 (continued)

(c) Captain America is standing 60 metres from the base of a building. He is watching Spiderman climb up the side of the building at a constant rate of 15 m/s. **3**



The angle of elevation from Captain America to Spiderman is θ. How fast is this angle increasing 6 seconds after $\theta = 0$? (Give your answer in radians/second).

(d) A bag contains 5 balls numbered from 1 to 5. The chance of choosing each ball is the same. One ball is chosen at random and its number is noted. The ball is then returned to the bag. This is done a total of five times and the numbers noted are recorded in order.

(i) In how many ways can the five balls be drawn if there are no restrictions? **1**

(ii) What is the probability that each ball is selected exactly once? **1**

(iii) What is the probability that exactly one of the balls is not selected? **2**

End of Question 13

Question 14 (15 marks) Answer on the appropriate page

(a) The velocity of a particle moving along the x-axis is given by $v = 2(x+1)^2$ m/s. **2**

Find the acceleration of the particle at $x = 2$.

(b) Use mathematical induction to prove that for all integers $n \geq 1$, **3**

$4^n + 15n - 1$ is divisible by 9.

(c) A survey was conducted at Baulkham Hills High School to decide whether a new menu for the canteen should be implemented. Staff responses were 23% of the total responses and the remaining 77% were from the students.
It is known that 31% of the staff who responded were in favour of changing the menu and 58% of students were in favour of changing the menu.

Assuming a very large number of responses were received:

(i) Find the probability that a randomly chosen person is in favour of changing the menu. **1**

(ii) If 5 survey responses were selected at random, find the probability that the majority of the responses were in favour of changing the menu. **2**

(d) By considering the expansion of $x(1+x)^n$, prove $\displaystyle\sum_{r=0}^{n}(r+1)\ {}^{n}\mathbf{C}_{r} = 2^n\left(\frac{n}{2}+1\right)$ **3**

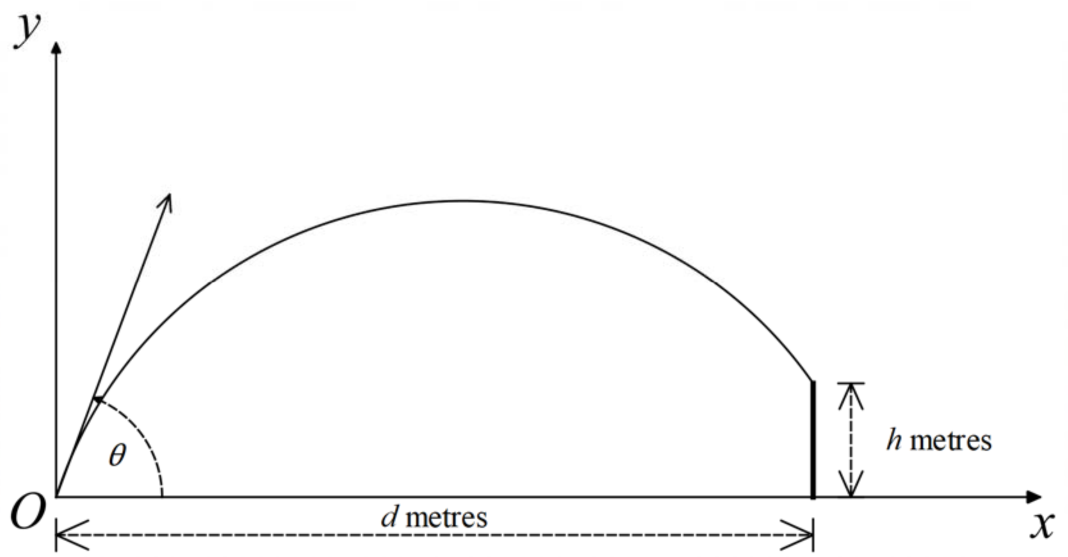
Question 14 continues on the following page

Question 14 (continued)

(e) A ball is projected from level ground at an angle of θ and a velocity of V m/s. You may assume the equations of motion are $x = Vt\cos\theta$ and $y = -\frac{gt^2}{2} + Vt\sin\theta$ (DO NOT PROVE THESE RESULTS).

(i) Show that the maximum height achieved by the ball is $\frac{V^2\sin^2\theta}{2g}$. **2**

(ii) The ball just clears a wall of height h metres that is d metres from the point of projection. **2**



Show that the greatest height reached is $\frac{d^2\tan^2\theta}{4(d\tan\theta - h)}$.

End of paper

2019 EXTENSION 1 TRIAL SOLUTIONS

① D:
$$\tan\left(\frac{\pi}{4}+x\right) = \frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}$$
$$= \frac{1+\tan x}{1-\tan x} \quad \begin{matrix}\times\cos x\\ \times\cos x\end{matrix}$$
$$= \frac{\cos x+\sin x}{\cos x-\sin x}$$

② B
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$
$$= \frac{2t}{1}$$
when $t=-3$, $\frac{dy}{dx}=-6$, $y=11$, $x=-6$
$$y-11 = -6(x+6)$$
$$y-11 = -6x-36$$
$$6x+y+25$$

③ : A

when $t=0$, $N=400+100$
$$=500$$
as $t\to\infty$, $N\to 400+100\times 0$
$$\therefore N\to 400$$
Difference $= 500-400$
$$=100$$

④ A
$$2\sin x\cos\alpha + 2\cos x\sin\alpha = \sin x-\sqrt{3}\cos x$$
Equating $2\cos\alpha = 1$ $\qquad 2\sin\alpha = -\sqrt{3}$
$\therefore$ 4th quadrant
$$\therefore \alpha = -\frac{\pi}{3}$$

⑤ A

General term $= {}^7C_k (x^2)^{7-k}(2x^{-1})^k$
$$= 2^k\cdot {}^7C_k\, x^{14-2k}\, x^{-k}$$
$$= 2^k\, {}^7C_k\, x^{14-3k}$$
For x^5: $\quad 14-3k=5$
$$9=3k$$
$$k=3$$
$\therefore$ Term in x^5 is $2^3\times {}^7C_3$

(6) B D: $-1 \le 2x \le 1$ R: $-\frac{\pi}{2} \le \frac{y}{3} \le \frac{\pi}{2}$

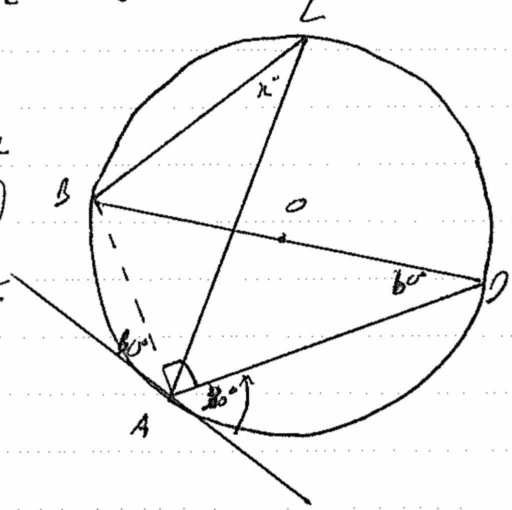
$-\frac{1}{2} \le x \le \frac{1}{2}$ $-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$

(7) C $\angle BAD = 90°$ ($\angle$ in a semicircle)

$\angle BAE = 60°$ ($\angle$ sum of straight line

$\angle ADB = 60°$ ($\angle$ in alternate segment)

$\angle ACB = x = 60°$ ($\angle$ at circumference standing on same arc)



(8) B $x = 2\cos(nt)$

$\dot{x} = -2n\sin(nt)$

when $x = 0$ $0 = 2\cos(nt)$

$nt = \frac{\pi}{2}$

sub in $\dot{x}$, $\sqrt{2} = \left|-2n \sin(\frac{\pi}{2})\right|$

$\sqrt{2} = 2n$

$n = \frac{1}{\sqrt{2}}$

(9) D $(\sin x + 2\cos x)(\sin x + \cos x) = 0$

$\sin x + 2\cos x = 0$ $\sin x + \cos x = 0$

$\sin x = -2\cos x$ $\sin x = -\cos x$

$\tan x = -2$ $\tan x = -1$

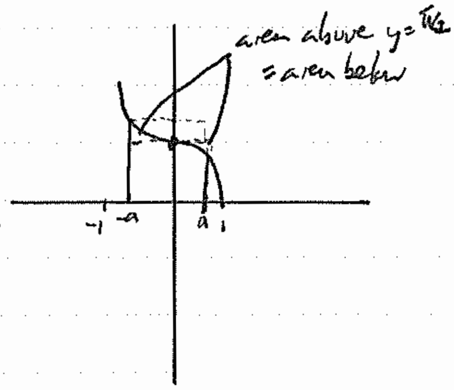
$\Downarrow$ $\Downarrow$

2 solutions 2 solutions

(10) C

Since area between $y = \cos^{-1} x$ and $y = \frac{\pi}{2}$ when $-a < x < 0$ is equal to area between $y = \frac{\pi}{2}$ and $y = \cos^{-1} x$ for $0 < x < a$ then a rectangle can be formed with area $2a \times \frac{\pi}{2}$

$\therefore$ Area $= a\pi$



11 a) $\int_0^3 \frac{dx}{\sqrt{9-x^2}} = \left[\sin^{-1}\frac{x}{3}\right]_0^3$

$= \sin^{-1} 1 - \sin^{-1} 0$

$= \frac{\pi}{2}$

(2) correct answer
(1) correct integral

b) $A(-2,1) \quad B(1:5)$

$-5 : 2$

$Q: \left(\frac{-4-5}{-5+2}, \frac{2-25}{-5+2}\right)$

$= \frac{-9}{-3}, \frac{-23}{-3}$

$= \left(3, 7\frac{2}{3}\right)$

(2) correct solution
(1) either x or y value correct
(1) correct internal division

c) $\tan\theta = 2\sin\theta\cos\theta$

$\sin\theta = 2\sin\theta\cos^2\theta$

$2\sin\theta\cos^2\theta - \sin\theta = 0$

$\sin\theta(2\cos^2\theta - 1) = 0$

$\sin\theta = 0$ or $\cos^2\theta = \frac{1}{2}$

$\theta = 0, \pi \qquad \cos\theta = \pm\frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

$\therefore \theta = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi$

(3) correct solution
(2) at least 2 correct solutions
(1) converts to expressions in t
or uses $2\sin\theta\cos\theta = \sin 2\theta$
and attempts to solve equation

d) $y = \ln(\tan^{-1}(2x))$

$\frac{dy}{dx} = \frac{\frac{1}{1+(2x)^2} \times 2}{\tan^{-1}(2x)}$

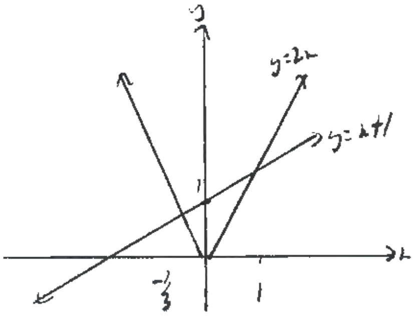
$= \frac{2}{(4x^2+1)\tan^{-1}(2x)}$

(3) correct solution
(2) correctly applies 2 of the 3 points below
(1) correctly applies 1 of the 3 points below
- recognises derivative of log $\ln x$
- recognises derivative of $\tan^{-1}x$
- recognises need to multiply by 2 (using chain rule)

11 e) common ratio $= \dfrac{\frac{2x}{x+1}}{1}$

to have a limiting sum $\left|\dfrac{2x}{x+1}\right| < 1 \quad x \neq -1$

$|2x| < |x+1|$



$2x = x+1$
$x = 1$

$-2x = x+1$
$-3x = 1$
$x = -\frac{1}{3}$

$\therefore \begin{cases} -\frac{1}{3} < x < 1, \text{ excluding } x = 0 \\ -\frac{1}{3} < x < 0 \text{ or } 0 < x < 1 \end{cases}$

(3) correct solution
(2) identifies $x = -\frac{1}{3}$ and x
(2) obtains $-1 < r < 1$
(ie omits absolute value)
(1) identifies $x = 1$

11 f) $\tan 3\theta = \frac{1}{\sqrt{3}}$

$3\theta = n\pi + \tan^{-1}\frac{1}{\sqrt{3}}$, where n is an integer

$3\theta = n\pi + \frac{\pi}{6}$

$\theta = \frac{n\pi}{3} + \frac{\pi}{18}$ or $\frac{\pi(6n+1)}{18}$ where n is an integer

(2) correct solution (must state n is integer)
(1) finds $\frac{\pi}{6}$

12 a) $8y = 3x - 7$

$y = \frac{3x}{8} - \frac{7}{8} \qquad y = -2x + 5$

$\tan\theta = \left|\dfrac{\frac{3}{8} - (-2)}{1 - (2 \times \frac{3}{8})}\right|$

$\tan\theta = 9.5$

$\theta = \tan^{-1} 9.5$

$\theta = 83.99\ldots$

$\theta = 84^\circ$ (nearest degree)

(2) correct solution
(1) substitutes gradients into formula

12b) $\int_1^2 x\sqrt{2+x^2}\,dx$

let $u = 2+x^2$
$du = 2x\,dx$
when $x=2,\ u=6$
$x=1,\ u=3$

(3) correct solution
(2) correct integral in terms of u
(1) correct integrand in terms of u

$$\int_3^6 \sqrt{u}\ \frac{du}{2}$$

$$= \frac{1}{2}\int_3^6 u^{1/2}\,du$$

$$= \frac{1}{2}\left[\frac{2u^{3/2}}{3}\right]_3^6$$

$$= \frac{1}{3}\left(6^{3/2} - 3^{3/2}\right)$$

$$= \frac{1}{3}\left(6\sqrt{6} - 3\sqrt{3}\right)$$

OR $2\sqrt{6} - \sqrt{3}$

OR $\sqrt{3}\left(2\sqrt{2}-1\right)$

OR 3.1669 (4dp)

12 c) $\alpha+\beta+\gamma = 2$

$\alpha\beta+\alpha\gamma+\beta\gamma = 4$

$\alpha\beta\gamma = -7$

(2) correct solution
(1) finds $\Sigma\alpha$, $\Sigma\alpha\beta$ and $\Sigma\alpha\beta\gamma$
(1) expresses $(2\alpha+1)(2\beta+1)(2\gamma+1)$ in terms of $\Sigma\alpha$, $\Sigma\alpha\beta$, $\Sigma\alpha\beta\gamma$

$$(2\alpha+1)(2\beta+1)(2\gamma+1)$$

$$= 8\alpha\beta\gamma + 4(\alpha\beta+\alpha\gamma+\beta\gamma) + 2\alpha+2\beta+2\gamma+1$$

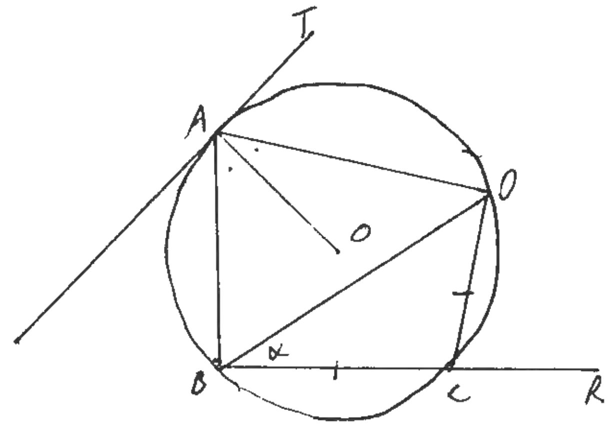
$$= 8\times -7 + 4\times 4 + 2(\alpha+\beta+\gamma)+1$$

$$= -56 + 2\times 2 + 16 + 1$$

$$= -35$$

12 d) next page

12 d)



① correct solution (including diagram)

i) Prove: $\angle OCR = 2\alpha$

$\angle BCD = \alpha$ (equal sides opposite equal $\angle$'s in isosceles $\triangle BCD$)

$\angle DCR = 2\alpha$ (exterior $\angle$ opposite of $\triangle BCD$)

ii) $\angle BAD = 2\alpha = \angle DCR$ (exterior $\angle$ of cyclic quadrilateral BADC) equals interior opposite angle

① correct solution

$\angle BAO = \angle DAO = \frac{1}{2}\angle BAD$

$\therefore \angle OAD = \alpha$

iii) $\angle TAD = \frac{\pi}{2} - \alpha$ (radius $\perp$ tangent at A)

② correct solution
① finds $\angle TAD$ with reason.

$\angle ABD = \frac{\pi}{2} - \alpha$ (alternate segment theorem)

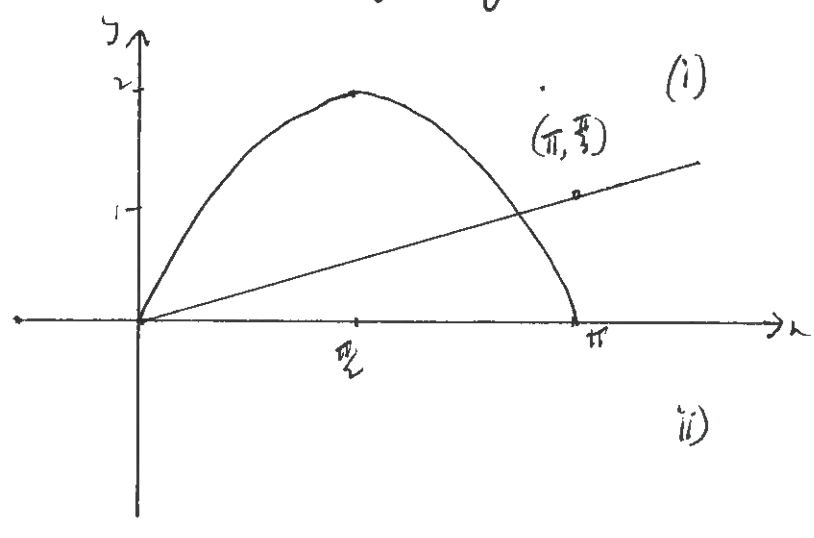
$\angle ABC = \angle ABD + \angle DBC$

$= \frac{\pi}{2} - \alpha + \alpha$

$= \frac{\pi}{2}$

$\therefore \angle ABC$ is a right angle

12 e)



(i) ① correct solution

ii) ① correct solution

12e(iii) $y = 2\sin x$

$y = \frac{x}{3}$

let $f(x) = 2\sin x - \frac{x}{3}$

$f'(x) = 2\cos x - \frac{1}{3}$

$x_1 = 2\cdot7 - \dfrac{2\sin 2\cdot7 - \frac{2\cdot7}{3}}{2\cos 2\cdot7 - \frac{1}{3}}$

$= 2\cdot67887\ldots$

$= 2\cdot679$ (3 dp)

(2) correct solution

(1) correct expression for approximation

(1) used $f(x) = \sin x$ to obtain $3\cdot173$

13 a) (ii) $y = 5 - \sqrt{x}$ $x \geq 0,\ y \leq 5$

inverse $x = 5 - \sqrt{y}$ $x \leq 5,\ y \geq 0$

$\sqrt{y} = 5 - x$

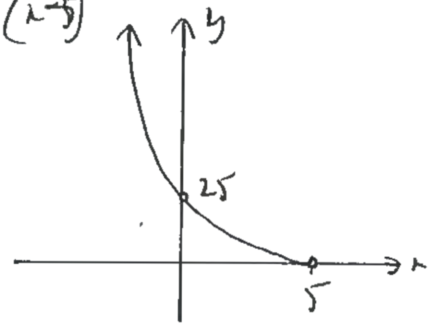
$y = (5 - x)^2$ where $x \leq 5$

or $y = (x-5)^2$

(2) correct solution

(1) expression with x and y interchanged (ignore $x \leq 5$)

(i)



(1) correct solution

b)(i) At P, gradient of tangent $= p$

$\therefore$ equation of k: $y = px + a$

(2) correct solution

(1) identifies gradient of k or y intercept of k

ii) when $y = 0$ $0 = px + a$

$px = -a$

$x = \frac{-a}{p}$

$\therefore Q$ is $\left(\frac{-a}{p}, 0\right)$ $S(0, a)$

Midpoint M of QS: $M = \left(\dfrac{\frac{-a}{p} + 0}{2}, \dfrac{a + 0}{2}\right)$

$M = \left(\frac{-a}{2p}, \frac{a}{2}\right)$

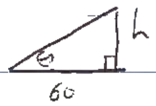
(2) correct solution

(1) finds coordinates of Q

iii) Locus of M: $y = \frac{a}{2}$ (as a is constant)

(1) correct answer

13 c) Let h metres be the height of spiderman above the horizontal



$\tan\theta = \frac{h}{60}$

$h = 60\tan\theta$

$\frac{dh}{d\theta} = 60\sec^2\theta$, $\frac{dh}{dt} = 15$ $\frac{d\theta}{dt} = ?$

$$\frac{dh}{dt} = \frac{dh}{d\theta} \times \frac{d\theta}{dt}$$

$$15 = 60(\tan^2\theta + 1)\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{4\left[\left(\frac{90}{60}\right)^2 + 1\right]}$$

$$\frac{d\theta}{dt} = \frac{1}{13} \text{ radians/second}$$

(3) correct solution
(2) establishes chain rule with either $\frac{d\theta}{dh}, \frac{dh}{dt}, \frac{d\theta}{dt}$ etc
(1) finds $\frac{d\theta}{dh}$ or $\frac{dh}{d\theta}$

13d) (i) $5^5 = 3125$ (1) correct answer

ii) $P(\text{all different}) = 1 \times \frac{4}{5} \times \frac{3}{5} \times \frac{2}{5} \times \frac{1}{5}$ (1) correct answer

$= \frac{24}{625}$

iii) $P(\text{exactly 1 not selected}) = 5 \times 4 \times \frac{\frac{5!}{2!}}{5^5}$

which letter gets repeated

which letter gets ommitted

(2) correct solution
(1) progress towards ways of arranging (eg attempts to 5 objects with 2 alike find one possibility 11234)

14 a) $v = 2(x+1)^2$

$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dx}\left(\frac{1}{2} \times 4(x+1)^4\right)$

$= 2 \times 4(x+1)^3$

$a = 8(x+1)^3$

when $x = 2$, $a = 8 \times 3^3$

$\therefore a = 216 \text{ ms}^{-2}$

(2) correct solution
(1) attempts to differentiate using $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

b) Test $n = 1$

$4^1 + 15 \times 1 - 1$

$= 18$

$= 2 \times 9$

$\therefore$ True for $n = 1$

Assume true for $n = k$

$4^k + 15k - 1 = 9P$ where P is an integer

For $n = k+1$ we wish to prove

$4^{k+1} + 15(k+1) - 1 = 9Q$ where Q is an integer

(3) correct solution
(2) 3 of the four steps
(1) 2 of the four steps

"Four steps"
(1) Proves true for $n=1$
(2) Assumes true for $n=k$
(3) Proves true for $n=k+1$
(4) Logic conclusion

$4^{k+1} + 15(k+1) - 1$

$= 4.4^k + 15k + 14$

$= 4(9P - 15k + 1) + 15k + 14$ using assumption

$= 9 \times 4P - 45k + 18$

$= 9(4P - 5k + 2)$

$= 9Q$ where Q is an integer since P and k are integers

$\therefore$ If true for $n = k$ it is true for $n = k+1$. But it is true for $n = 1$, $\therefore$ true for $n = 1+1 = 2, 3, 4$ and so on for all positive integers $n \geq 1$.

14 c) i) P (in favour of changing)

$= P(\text{staff \& in favour}) \text{ OR } P(\text{student in favour})$

$= 0.23 \times 0.31 + 0.77 \times 0.58$

$= 0.5179$ or 51.79%

① correct answer

ii) P (majority of 5 in favour)

$= P(3 \text{ or } 4 \text{ or } 5 \text{ in favour})$

$= \binom{5}{3}(0.5179)^3(0.4821)^2 + \binom{5}{4}(0.5179)^4(0.4821)^1 + \binom{5}{5}(0.5179)^5$

$= 53.35\%$ (2 decimal places)

② correct solution
① attempts to use binomial probability for at least one case

14 d) $x(1+x)^n = x\left[\binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{k}x^k + \dots \binom{n}{n}x^n\right]$

$x(1+x)^n = \binom{n}{0}x + \binom{n}{1}x^2 + \binom{n}{2}x^3 + \dots + \binom{n}{k}x^{k+1} + \dots + \binom{n}{n}x^{n+1}$

Differentiate w.r.t. x.

$(1+x)^n \cdot 1 + x\,n(1+x)^{n-1} = \binom{n}{0} + 2\binom{n}{1}x + 3\binom{n}{2}x^2 + \dots + (k+1)\binom{n}{k}x^k + \dots + (n+1)\binom{n}{n}x^n$

Let $x = 1$

$2^n + n\,2^{n-1} = \binom{n}{0} + 2\binom{n}{1} + 3\binom{n}{2} + \dots + (k+1)\binom{n}{k} + \dots + (n+1)\binom{n}{n}$

$2^n\left[1 + \frac{n}{2}\right] = \binom{n}{0} + 2\binom{n}{1} + 3\binom{n}{2} + \dots (k+1)\binom{n}{k} + \dots + (n+1)\binom{n}{n}$

$$\sum_{r=0}^{n} (r+1)\binom{n}{r} = 2^n\left(\frac{n}{2} + 1\right)$$

③ correct solution
② significant progress (eg expands and differentiates)
① Limited progress (eg expands/substitutes $x=1$)

14 e) i) $x = Vt\cos\theta$ $\quad y = -\frac{gt^2}{2} + Vt\sin\theta$

$\dot{y} = -gt + V\sin\theta$

At max height $\dot{y} = 0$

$0 = -gt + V\sin\theta$

$gt = V\sin\theta$

$t = \frac{V\sin\theta}{g}$

sub in y, $\quad y = \frac{-g}{2}\,\frac{V^2\sin^2\theta}{g^2} + V\sin\theta\,\frac{V\sin\theta}{g}$

Max height $= \frac{V^2\sin^2\theta}{2g}$

② correct solution
① finds time to greatest height

14 e (ii) when $x = d$, $y = h$

(2) correct solution
(1) substitutes d and h into equation of motion and solve simultaneously with

$$d = Vt\cos\theta$$
$$t = \frac{d}{V\cos\theta}$$

$$h = -\frac{gt^2}{2} + Vt\sin\theta$$
$$h = \frac{-g d^2}{2V^2\cos^2\theta} + \frac{Vd}{V\cos\theta}\sin\theta$$

$$h = \frac{-gd^2}{2V^2\cos^2\theta} + d\tan\theta$$

$$\frac{gd^2}{2V^2\cos^2\theta} = d\tan\theta - h$$

$$\frac{g}{V^2} = \frac{2(d\tan\theta - h)\cos^2\theta}{d^2}$$

$$\frac{V^2}{g} = \frac{d^2}{2(d\tan\theta - h)\cos^2\theta}$$

sub in (i) Max height $= \frac{d^2}{2(d\tan\theta - h)\cos^2\theta} \times \frac{\sin^2\theta}{2}$

$$= \frac{d^2\tan^2\theta}{4(d\tan\theta - h)}$$